College Preparatory Mathematics – Core Connections Course 3
Volume 1 MATH NOTES

CHAPTER 1

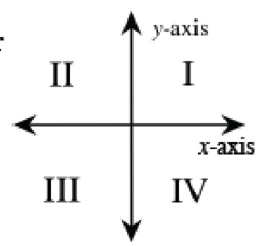
MATH NOTES

METHODS AND MEANINGS

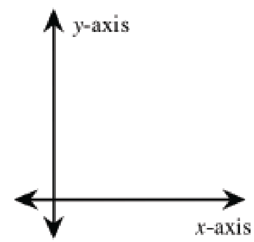
Axes, Quadrants, and Graphing on an xy-Coordinate Graph

Coordinate axes on a flat surface are formed by drawing vertical and horizontal number lines that meet at 0 on each number line and form a right angle (90°). The x- and y-**axes** help define points on a graph (called a "Cartesian Plane"). The x**-axis** is horizontal, while the y**-axis** is vertical. The x- and y-axes divide the graphing area into four sections called **quadrants**.

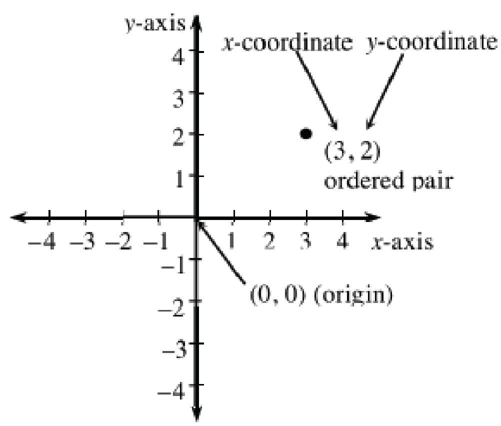
4-quadrant graph:



1st-quadrant graph:



Numerical data can be graphed on a plane using **points**. Points on the graph are identified by two numbers in an **ordered pair** written as (x, y). The first number is the x**-coordinate** of the point, and the second number is the y**-coordinate**. The point $(0, 0)$ is called the **origin**.



To locate the point $(3, 2)$ on an xy-coordinate graph, go three units from the origin to the right to 3 on the horizontal axis and then, from that point, go 2 units up (using the y-axis scale). To locate the point $(-2, -4)$, go 2 units from the origin to the left to -2 on the horizontal axis and then 4 units down (using the y-axis scale).

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Writing Equations using the 5-D Process

The 5-D Process is an organized method to help write equations and solve problems. The D's stand for Describe/Draw, Define, Do, Decide, and Declare. An example of this work is shown below.

Example Problem: The base of a rectangle is 13 centimeters longer than the height. If the perimeter is 58 centimeters, find the base and the height of the rectangle.

Describe/Draw: The shape is a rectangle, and we are looking at the perimeter.

	Define		Do	Decide
	Height (trial)	Base (height + 13)	Perimeter 2(base) + 2(height)	58?
Trial 1:	10	$10 + 13 = 23$	$2(23) + 2(10) = 66$	66 is too high

Use any trial value.

Use the relationships stated in the problem to determine the values of the other quantities (such as base and perimeter).

Now use the trial to create an equation by defining and adding a variable line.

Let x represent the height in cm	x	$x + 13$	$2(x) + 2(x + 13)$	$2(x) + 2(x + 13) = 58$

Now use your algebra skills to solve the equation.

$$2(x) + 2(x + 13) = 58$$
$$2x + 2x + 26 = 58$$
$$4x + 26 = 58$$
$$4x = 32$$
$$x = 8$$

Declare: The base is 21 centimeters, and the height is 8 centimeters.

If you do not write an equation, you can solve the problem by making more trials until you find the answer.

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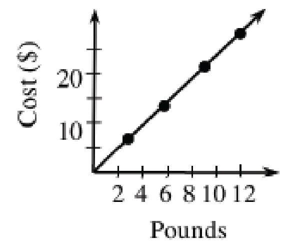
Proportional Relationships

A **proportional relationship** can be seen in a table: if one quantity is multiplied by an amount, the corresponding quantity is multiplied by the same amount. On a graph, a proportional relationship is linear and goes through the origin.

Proportional example: Three pounds of chicken costs $7.00. Below, other values are shown in the table and plotted on the graph.

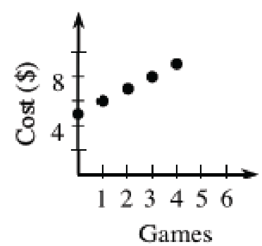
Pounds	0	3	6	9	12
Cost ($)	0	7	14	21	28



The relationship between pounds and cost is proportional.

Non-proportional example: The video arcade costs $5.00 to enter and $1.00 per game.

Games	0	1	2	3	4
Cost ($)	5	6	7	8	9



The relationship between games and cost is *not* proportional. For example, someone who plays four games ($9) does not pay twice as much as someone who played two games ($7). There is no multiplier for the relationship. The graph does not go through the origin.

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Solving Proportions

If a relationship is known to be proportional, ratios from the situation are equal. An equation stating that two ratios are equal is called a **proportion**. Some examples of proportions are:

$$\frac{6\text{ mi}}{2\text{ hr}} = \frac{9\text{ mi}}{3\text{ hr}} \qquad \frac{5}{7} = \frac{50}{70}$$

Setting up a proportion is one strategy for solving for an unknown part of one ratio. For example, if the ratios $\frac{9}{2}$ and $\frac{x}{16}$ are equal, setting up the proportion $\frac{x}{16} = \frac{9}{2}$ allows you to solve for x.

Giant One: One way to solve this proportion is by using a Giant One to find the equivalent ratio. In this case, since 16 is 2 times 8, you create the Giant One shown at right.

$$\frac{x}{16} = \frac{9}{2} \cdot \frac{8}{8}$$
$$\frac{x}{16} = \frac{9 \cdot 8}{2 \cdot 8}$$
$$\frac{x}{16} = \frac{72}{16}$$

which shows that $x = 72$.

Undoing Division: Another way to solve the proportion is to think of the ratio $\frac{x}{16}$ as, "x divided by 16." To solve for x, use the inverse operation of division, which is multiplication. Multiplying both sides of the proportional equation by 16 "undoes" the division.

$$\frac{x}{16} = \frac{9}{2}$$
$$\left(\frac{16}{1}\right)\frac{x}{16} = \frac{9}{2}\left(\frac{16}{1}\right)$$
$$x = \frac{144}{2}$$
$$x = 72$$

Cross-Multiplication: This method of solving the proportion is a shortcut for using a Fraction Buster (multiplying each side of the equation by the denominators).

Fraction Buster

$$\frac{x}{16} = \frac{9}{2}$$
$$2 \cdot 16 \cdot \frac{x}{16} = \frac{9}{2} \cdot 2 \cdot 16$$
$$2 \cdot x = 9 \cdot 16$$
$$2x = 144$$
$$x = 72$$

Cross-Multiplication

$$\frac{x}{16} = \frac{9}{2}$$
$$\frac{x}{16} \quad \frac{9}{2}$$
$$2 \cdot x = 9 \cdot 16$$
$$2x = 144$$
$$x = 72$$

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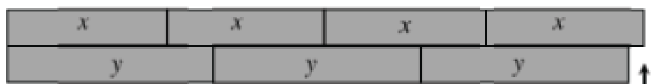
Non-Commensurate

Two measurements are called **non-commensurate** if no combination of one measurement can equal a combination of the other. For example, your algebra tiles are called non-commensurate because no combination of unit squares will ever be exactly equal to a combination of x-tiles (although at times they may appear close in comparison). In the same way, in the example below, no combination of x-tiles will ever be exactly equal to a combination of y-tiles.



No matter what number of each size tile, these two piles will never exactly match.

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Mathematics Vocabulary

Variable: A letter or symbol that represents one or more numbers.

Expression: A combination of numbers, variables, and operation symbols. For example, $2x+3(5-2x)+8$. Also, $5-2x$ is a smaller expression within the larger expression.

Term: Parts of the expression separated by addition and subtraction. For example, in the expression $2x+3(5-2x)+8$, the three terms are $2x$, $3(5-2x)$, and 8. The expression $5-2x$ has two terms, 5 and $-2x$.

Coefficient: The numerical part of a term. In the expression $2x+3(5-2x)+8$, for example, 2 is the coefficient of $2x$. In the expression $7x-15x^2$, both 7 and 15 are coefficients.

Constant term: A number that is not multiplied by a variable. In the expression $2x+3(5-2x)+8$, the number 8 is a constant term. The number 3 is not a constant term, because it is multiplied by a variable inside the parentheses.

Factor: Part of a multiplication expression. In the expression $3(5-2x)$, 3 and $5-2x$ are factors.

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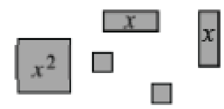
Combining Like Terms

Combining tiles that have the same area to write a simpler expression is called **combining like terms**. See the example shown at right.



$x^2 + 2x + 2$

When you are not working with actual tiles, it can help to picture the tiles in your mind. You can use these images to combine the terms that are the same. Here are two examples:

Example 1: $2x^2 + xy + y^2 + x + 3 + x^2 + 3xy + 2 \quad \Rightarrow \quad 3x^2 + 4xy + y^2 + x + 5$

Example 2: $3x^2 - 2x + 7 - 5x^2 + 3x - 2 \quad \Rightarrow \quad -2x^2 + x + 5$

A **term** is an algebraic expression that is a single number, a single variable, or the product of numbers and variables. The simplified algebraic expression in Example 2 above contains three terms. The first term is $-2x^2$, the second term is x, and the third term is 5.

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Evaluating Expressions and the Order of Operations

To **evaluate** an algebraic expression for particular values of the variables, replace the variables in the expression with their known numerical values and simplify. Replacing variables with their known values is called **substitution**. An example is provided below.

Evaluate $4x-3y+7$ for $x=2$ and $y=1$.

Replace x and y with their known values of 2 and 1, respectively, and simplify.

$$\begin{aligned} &4(2)-3(1)+7 \\ &=8-3+7 \\ &=12 \end{aligned}$$

When evaluating a complex expression, you must remember to use the **Order of Operations** that mathematicians have agreed upon. As illustrated in the example below, the order of operations is:

Original expression:

$$(10-3\cdot 2)\cdot 2^2-\frac{13-3^2}{2}+6$$

Circle expressions that are grouped within parentheses or by a fraction bar:

$$(10-3\cdot 2)\cdot 2^2-\frac{13-3^2}{2}+6$$

Simplify *within* circled terms using the order of operations:

$$(10-3\cdot 2)\cdot 2^2-\frac{13-3\cdot 3}{2}+6$$

- Evaluate exponents.

$$(10-6)\cdot 2^2-\frac{13-9}{2}+6$$

- Multiply and divide from left to right.

$$(4)\cdot 2^2-\frac{4}{2}+6$$

- Combine terms by adding and subtracting from left to right.

$$4\cdot 2^2-\frac{4}{2}+6$$

Circle the remaining terms:

$$4\cdot 2\cdot 2-\frac{4}{2}+6$$

Simplify *within* circled terms using the Order of Operations as described above.

$$16-2+6$$

$$20$$

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Commutative Properties

The **Commutative Property of Addition** states that when *adding* two or more number or terms together, order is not important. That is:

$a+b=b+a$ For example, $2+7=7+2$

The **Commutative Property of Multiplication** states that when *multiplying* two or more numbers or terms together, order is not important. That is:

$a \cdot b=b \cdot a$ For example, $3 \cdot 5=5 \cdot 3$

However, *subtraction* and *division* are <u>not</u> commutative, as shown below.

$$7-2 \neq 2-7 \text{ since } 5 \neq -5$$

$$50 \div 10 \neq 10 \div 50 \text{ since } 5 \neq 0.2$$

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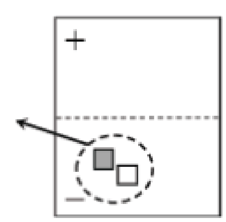
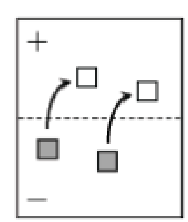
Simplifying an Expression ("Legal Moves")

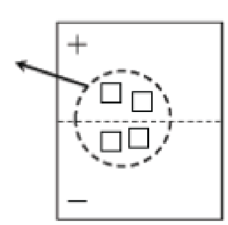
Three common ways to simplify or alter expressions on an Expression Mat are illustrated below.



- Removing an equal number of opposite tiles that are in the same region. For example, the positive and negative tiles in the same region at right combine to make zero.



- Flipping a tile to move it out of one region into the opposite region (i.e., finding its opposite). For example, the tiles in the "–" region at right can be flipped into the "+" region.



- Removing an equal number of identical tiles from both the "–" and the "+" regions. This strategy can be seen as a combination of the two methods above, since you could first flip the tiles from one region to another and then remove the opposite pairs.

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Associative and Identity Properties

The **Associative Property of Addition** states that when *adding* three or more number or terms together, grouping is not important. That is:

$(a+b)+c=a+(b+c)$ For example, $(5+2)+6=5+(2+6)$

The **Associative Property of Multiplication** states that when *multiplying* three or more numbers or terms together, grouping is not important. That is:

$(a\cdot b)\cdot c=a\cdot(b\cdot c)$ For example, $(5\cdot 2)\cdot 6=5\cdot(2\cdot 6)$

However, *subtraction* and *division* are <u>not</u> associative, as shown below.

$$(5-2)-3\neq 5-(2-3) \text{ since } 0\neq 6$$

$$(20\div 4)\div 2\neq 20\div(4\div 2) \text{ since } 2.5\neq 10$$

The **Identity Property of Addition** states that adding zero to any expression gives the same expression. That is:

$a+0=a$ For example, $6+0=6$

The **Identity Property of Multiplication** states that multiplying any expression by one gives the same expression. That is:

$1\cdot a=a$ For example, $1\cdot 6=6$

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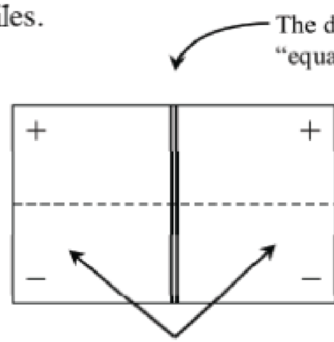
Using an Equation Mat

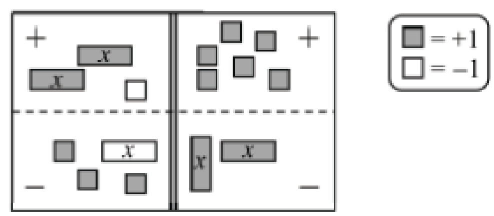
An **Equation Mat** can help you visually represent an equation with algebra tiles.

The double line represents the "equal" sign (=).



For each side of the equation, there is a positive and a negative region.

For example, the equation $2x-1-(-x+3)=6-2x$ can be represented by the Equation Mat at right. (Note that there are other possible ways to represent this equation correctly on the Equation Mat.)



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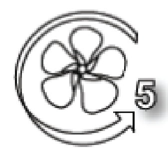
Patterns in Nature

Patterns are everywhere, especially in nature. One famous pattern that appears often is called the Fibonacci Sequence, a sequence of numbers that starts 1, 1, 2, 3, 5, 8, 13, 21, …

The Fibonacci numbers appear in many different situations in nature. For example, the number of petals on a flower is often a Fibonacci number, and the number of seeds on a spiral from the center of a sunflower is, too.

To learn more about Fibonacci numbers, search the Internet or check out a book from your local library. The next time you look at a flower, look for Fibonacci numbers!



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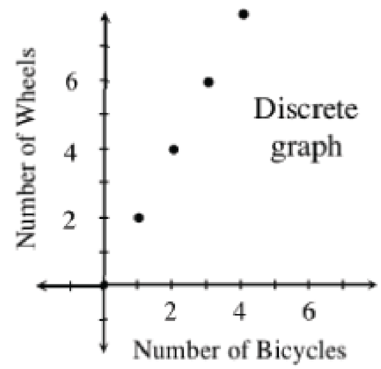
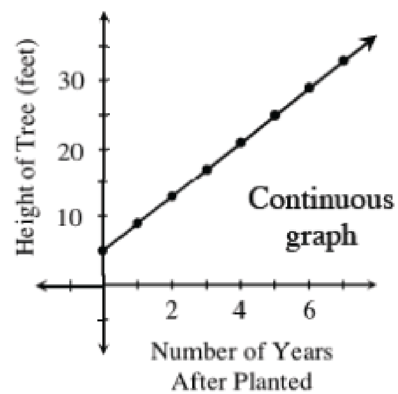
Discrete and Continuous Graphs

When a graph of data is limited to a set of separate, non-connected points, that relationship is called **discrete**. For example, consider the relationship between the number of bicycles parked at your school and the number of bicycle wheels. If there is one bicycle, it has two wheels. Two bicycles have four wheels, while three bicycles have six wheels. However, there cannot be 1.3 or 2.9 bicycles. Therefore, this data is limited because the number of bicycles must be a whole number, such as 0, 1, 2, 3, and so on.



When graphed, a discrete relationship looks like a collection of unconnected points. See the example of a discrete graph above.

When a set of data is not confined to separate points and instead consists of connected points, the data is called **continuous**. "John's Giant Redwood," problem 3-11, is an example of a continuous situation, because even though the table focuses on integer values of years (1, 2, 3, etc.), the tree still grows between these values of time. Therefore, the tree has a height at any non-negative value of time (such as 1.1 years after it is planted).



When data for a continuous relationship are graphed, the points are connected to show that the relationship also holds true for all points between the table values. See the example of a continuous graph above.

Note: In this course, tile patterns will represent elements of continuous relationships and will be graphed with a continuous line or curve.

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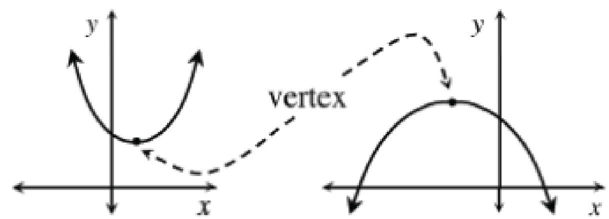
Parabolas

One kind of graph you will study in this class is called a **parabola**. Two examples of parabolas are graphed at right. Note that parabolas are smooth "U" shapes, not pointy "V" shapes.



The point where a parabola turns (the highest or lowest point) is called the **vertex**.

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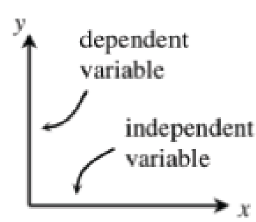
Independent and Dependent Variables

When one quantity (such as the height of a redwood tree) depends on another (such as the number of years after the tree was planted), it is called a **dependent variable**. That means its value is determined by the value of another variable. The dependent variable is usually graphed on the y-axis.

If a quantity, such as time, does not depend on another variable, it is referred to as the **independent variable**, which is graphed on the x-axis.

For example, in problem 3-46, you compared the amount of a dinner bill with the amount of a tip. In this case, the tip depends on the amount of the dinner bill. Therefore, the tip is the dependent variable, while the dinner bill is the independent variable.



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Complete Graph

A complete graph has the following components:

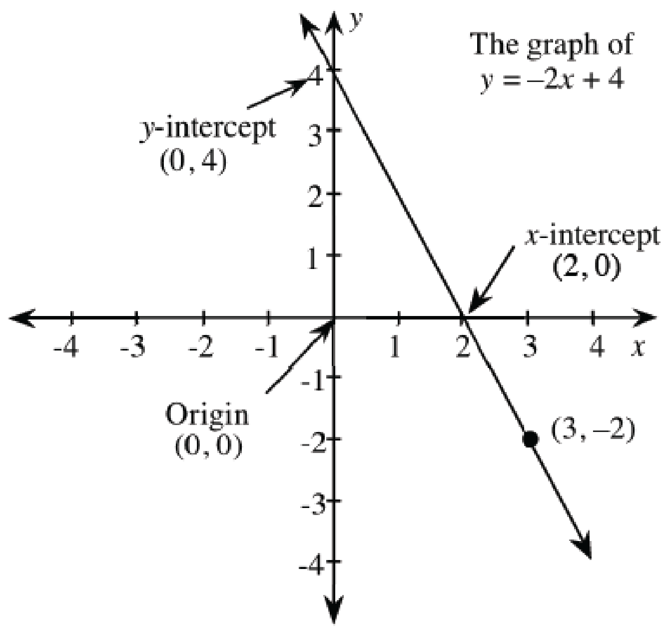
- x-axis and y-axis labeled, clearly showing the scale.
- Equation of the graph near the line or curve.
- Line or curve extended as far as possible on the graph.
- Coordinates of special points stated in (x, y) format.

x	–1	0	1	2	3
y	6	4	2	0	–2

Tables can be formatted horizontally, like the one above, or vertically, as shown below.

x	y
–1	6
0	4
1	2
2	0
3	–2
4	–4

Throughout this course, you will continue to graph lines and other curves. Be sure to label your graphs appropriately.



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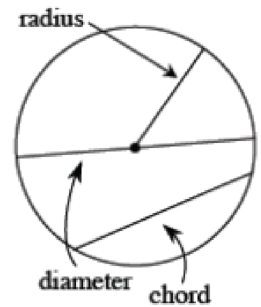
Circle Vocabulary, Circumference, and Area

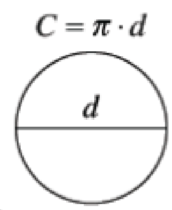


The **radius** of a circle is a line segment from its center to any point on the circle. The term is also used for the length of these segments. More than one radius are called **radii**.

A **chord** of a circle is a line segment joining any two points on a circle.

A **diameter** of a circle is a chord that goes through its center. The term is also used for the length of these chords. The length of a diameter is twice the length of a radius.

The **circumference** (C) of a circle is its perimeter, or the "distance around" the circle.



The number π (read "pi") is the ratio of the circumference of a circle to its diameter. That is, $\pi = \frac{\text{circumference}}{\text{diameter}}$. This definition is also used as a way of computing the circumference of a circle if you know the diameter, as in the formula $C = \pi d$ where C is the circumference and d is the diameter. Since the diameter is twice the radius ($d = 2r$), the formula for the circumference of a circle using its radius is $C = \pi(2r)$ or $C = 2\pi \cdot r$.

The first few digits of π are 3.141592.

To find the **area** (A) of a circle when given its radius (r), square the radius and multiply by π. This formula can be written as $A = r^2 \cdot \pi$. Another way the area formula is often written is $A = \pi \cdot r^2$.

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Solving a Linear Equation

When solving an equation like the one shown below, several important strategies are involved.

$$
\begin{array}{rcll}
3x-2+4 &=& x-6 & \\
3x+2 &=& x-6 & \text{combine like terms} \\
\underline{-x\quad} &=& \underline{-x\quad} & \text{subtract } x \text{ on both sides} \\
2x+2 &=& -6 & \\
\underline{-2} &=& \underline{-2} & \text{subtract 2 on both sides} \\
\dfrac{2x}{2} &=& \dfrac{-8}{2} & \text{divide both sides by 2} \\
x &=& -4 &
\end{array}
$$

- **Simplify.** Combine like terms and "make zeros" on each side of the equation whenever possible.
- **Keep equations balanced.** The equal sign in an equation indicates that the expressions on the left and right are balanced. Anything done to the equation must keep that balance.
- **Get x alone.** Isolate the variable on one side of the equation and the constants on the other.
- **Undo operations.** Use the fact that addition is the opposite of subtraction and that multiplication is the opposite of division to solve for x. For example, in the equation $2x=-8$, since the 2 and x are multiplied, dividing both sides by 2 will get x alone.

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Solutions to an Equation with One Variable

A **solution** to an equation gives a value of the variable that makes the equation true. For example, when 5 is substituted for x in the equation at right, both sides of the equation are equal. So $x=5$ is a solution to this equation.

$$4x-2=3x+3$$
$$4(5)-2=3(5)+3$$
$$18=18$$

An equation can have more than one solution, or it may have no solution. Consider the examples at right.

Notice that no matter what the value of x is, the left side of the first equation will never equal the right side. Therefore, you say that $x+2=x+6$ has **no solution**.

Equation with no solution:
$x+2=x+6$

However, in the equation $x-3=x-3$, no matter what value x has, the equation will always be true. All numbers can make $x-3=x-3$ true. Therefore, you say the solution for the equation $x-3=x-3$ is **all numbers**.

Equation with infinite solutions:
$x-3=x-3$

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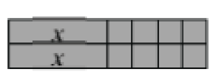
The Distributive Property

The **Distributive Property** states that for any three terms a, b, and c:

$$a(b+c)=ab+ac$$

That is, when a multiplies a group of terms, such as $(b+c)$, it multiplies *each* term of the group. For example, when multiplying $2(x+4)$, the 2 multiplies both the x and the 4. This can be represented with algebra tiles, as shown below.



$$2(x+4)=2\cdot x+2\cdot 4=2x+8$$

The 2 multiplies each term.

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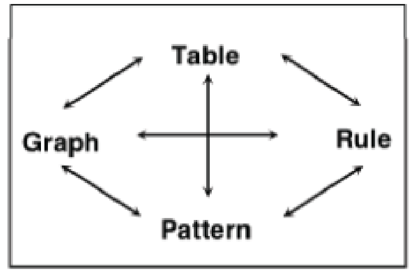
Representations of Patterns Web

Consider the **tile patterns** below. The number of tiles in each figure can also be represented in an ***x→y* table**, on a **graph**, or with a **rule** (equation).

Remember that in this course, tile patterns will be considered to be elements of continuous relationships and thus will be graphed with a continuous line or curve.



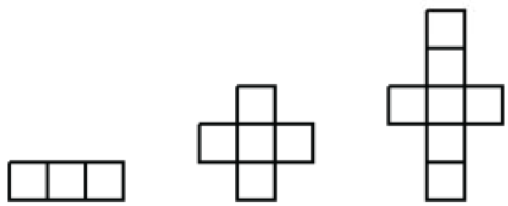
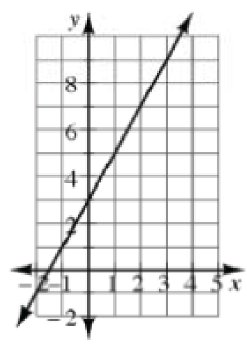


Figure 0 Figure 1 Figure 2

Tile Pattern



Graph

$$y = 2x + 3$$

Rule (Equation)

Figure Number (x)	0	1	2
Number of Tiles (y)	3	5	7

***x→y* Table**

METHODS AND MEANINGS

MATH NOTES

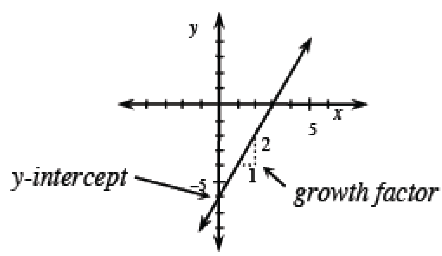
Linear Equations

A **linear equation** is an equation that forms a line when it is graphed. This type of equation may be written in several different forms. Although these forms look different, they are equivalent; that is, they all graph the same line.

Standard form: An equation in $ax + by = c$ form, such as $-6x + 3y = 18$.

$y = mx + b$ form: An equation in $y = mx + b$ form, such as $y = 2x + 6$.

You can quickly find the **growth factor** and ***y*-intercept** of a line in $y = mx + b$ form. For the equation $y = 2x - 6$, the growth factor is 2 , while the y-intercept is $(0, -6)$.



$$y = 2x - 6$$

growth factor *y-intercept*

METHODS AND MEANINGS

MATH NOTES

Equivalent Equations

Two equations are **equivalent** if they have all the same solutions. There are many ways to change one equation into a different, equivalent equation. Common ways include: *adding* the same number to both sides, *subtracting* the same number from both sides, *multiplying* both sides by the same number, *dividing* both sides by the same (non-zero) number, and *rewriting* one or both sides of the equation.

For example, the equations below are all equivalent to $2x + 1 = 3$:

$20x + 10 = 30$ $\qquad$ $2(x + 0.5) = 3$

$\frac{2x}{3} + \frac{1}{3} = 1$ $\qquad$ $0.002x + 0.001 = 0.003$

MATH NOTES

METHODS AND MEANINGS

Solving Equations with Fractions (also known as Fraction Busters)

Example: Solve $\frac{x}{3}+\frac{x}{5}=2$ for x.

$$\frac{x}{3}+\frac{x}{5}=2$$

This equation would be much easier to solve if it had no fractions. Therefore, the first goal is to find an equivalent equation that has no fractions.

The lowest common denominator of $\frac{x}{3}$ *and* $\frac{x}{5}$ *is* 15.

To eliminate the denominators, multiply both sides of the equation by the common denominator. In this example, the lowest common denominator is 15, so multiplying both sides of the equation by 15 eliminates the fractions. Another approach is to multiply both sides of the equation by one denominator and then by the other.

$$15\cdot\left(\frac{x}{3}+\frac{x}{5}\right)=15\cdot 2$$

$$15\cdot\frac{x}{3}+15\cdot\frac{x}{5}=15\cdot 2$$

Either way, the result is an equivalent equation without fractions:

$$5x+3x=30$$
$$8x=30$$

The number used to eliminate the denominators is called a **Fraction Buster**. Now the equation looks like many you have seen before, and it can be solved in the usual way.

$$x=\frac{30}{8}=\frac{15}{4}=3.75$$

$$\frac{3.75}{3}+\frac{3.75}{5}=2$$

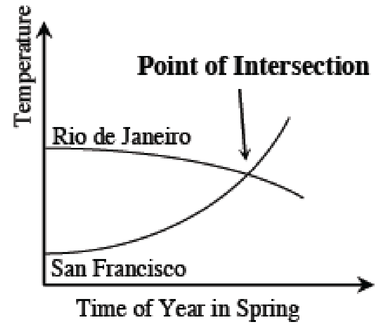
Once you have found the solution, remember to check your answer.

$$1.25+0.75=2$$

MATH NOTES

METHODS AND MEANINGS

Systems of Equations Vocabulary



The point where two lines or curves intersect is called a **point of intersection**. This point's significance depends on the context of the problem.

Two or more lines or curves used to find a point of intersection are called a **system of equations**. A system of equations can represent a variety of contexts and can be used to compare how two or more things are related. For example, the system of equations graphed above compares the temperature in two different cities over time.

MATH NOTES

METHODS AND MEANINGS

The Equal Values Method

The **Equal Values Method** is a non-graphing method to find the point of intersection or solution to a system of equations.

Start with two equations in $y = mx + b$ form, such as $y = -2x + 5$ and $y = x - 1$. Take the two expressions that equal y and set them equal to each other. Then solve this new equation to find x. See the example at right.

$$-2x + 5 = x - 1$$
$$-3x = -6$$
$$x = 2$$

Once you know the x-coordinate of the point of intersection, substitute your solution for x into *either* original equation to find y. In this example, the first equation is used.

$$y = -2x + 5$$
$$y = -2(2) + 5$$
$$y = 1$$

A good way to check your solution is to substitute your solution for x into *both* equations to verify that you get equal y-values.

$$y = x - 1$$
$$y = (2) - 1$$
$$y = 1$$

Write the solution as an ordered pair to represent the point on the graph where the equations intersect.

$$(2, 1)$$

Methods and Meanings

Math Notes

Solutions to a System of Equations

A **solution** to a system of equations gives a value for each variable that makes both equations true. For example, when 4 is substituted for x and 5 is substituted for y in both equations at right, both equations are true. So $x = 4$ and $y = 5$ or $(4, 5)$ is a solution to this system of equations. When the two equations are graphed, $(4, 5)$ is the point of intersection.

Some systems of equations have no solutions or infinite solutions. Consider the examples at right.

Notice that the Equal Values Method would yield $3 = 4$, which is never true. When the lines are graphed, they are parallel. Therefore, the system has **no solution**.

In the third set of equations, the second equation is just the first equation multiplied by 2. Therefore, the two lines are really the same line and have **infinite solutions**.

System with one solution: intersecting lines

$$x - y = -1$$
$$2x - y = 3$$

System with no solution: parallel lines

$$x + y = 3$$
$$x + y = 4$$

System with infinite solutions: coinciding lines

$$x + y = 3$$
$$2x + 2y = 6$$